## Polytopes and Beyond

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Grünbaum and Shephard remarked that there were three developments which foreshadowed the modern theory of convex polytopes.

- The publication of Euclid's Elements and the five Platonic solids in 300 BC.
- Euler's formula in a 1750 letter to Goldbach which states that f\_0 f\_1 + f\_2 = 2 holds for any 3-dimensional polytope, where f\_i is the number of i-dimensional faces.
- The discovery of polytopes in dimensions greater or equal to four by Schl\"afli in the 1850's.

We will use these as a springboard to describe the theory of convex polytopes leading into the 21st century and beyond. Our survey will include recent results for flag enumeration of polytopes, Bruhat graphs, balanced digraphs, Whitney stratified spaces and quasi-graded posets.