Diagonals of rational power series and their uses in combinatorics, number theory, and computer science

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Given a power series $F(x_1,\ldots x_d) = \sum f_{i_1,\,i_d} x_1^{i_1}\ldots x_d^{i_d}$, one can form a one-variable power series $\Delta(F)(t) = \sum f_{n,n,\ldots,n} t^n$, called the diagonal of F. When F is the power series expansion of a rational function, the diagonal enjoys of F enjoys many nice properties, including satisfying a linear homogeneous differential equation with polynomial coefficients. Many natural generating functions arising in combinatorial enumeration can be expressed as diagonals and this fact often gives one a wealth of information about congruences of coefficients mod primes and asymptotic information. We give a survey of the theory of diagonals and discuss some more recent results and some of their applications to other areas of mathematics.