

Crystal bases and rigged configurations

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In my talk I will report on the present status of our project to understand a certain identity, called $X=M$, that has arisen in the end of the 20th century from the studies of combinatorial aspects of quantum integrable systems. Both sides of $X=M$ are as simple as

$$\sum_{b \in \mathcal{P}(B, \lambda)} q^{E(b)} = \sum_{\nu \in C(L(B), \lambda)} q^{c(\nu)} \prod_{a, i} m^{(a)}_{i+p^{(a)}_i} \text{choose } m^{(a)}_{i-q}$$

but what it implies is surprisingly deep. For instance, it is related to the following topics.

1. Generalizing Lascoux-Schützenberger's charge and Schützenberger's involution to other root systems
2. Mysterious combinatorial bijection due to Kerov-Kirillov-Reshetikhin.
3. Calculating the number of irreducible modules in a tensor product of gl_n -modules of rectangular shapes.
4. Closed formula for a branching function corresponding to a pair of affine Lie algebra and its underlying finite-dimensional simple Lie algebra.
5. Linearizing a certain ultra-discrete nonlinear integrable system called box-ball system
6. Geometric crystals introduced by Berenstein-Kazhdan and a solution to the Yang-Baxter equation by positive birational maps.

For affine type A most (but not all!) topics are fairly well understood. However, apart from type A many conjectures are still waiting to be settled. For instance, item 2 of the above list was just worked out for type D only in this March. Taking this wonderful opportunity to talk at FPSAC meeting, I would like to persuade (especially young) people to join in this project.