The Smith normal form distribution of a random integer matrix

Yinghui Wang*[1] and Richard P. Stanley*[2]

[1]Department of Mathematics – Mathematics Department Columbia University Room 509, MC 4406 2990 Broadway New York, NY 10027, United States

[2]Department of Mathematics [MIT] – Massachusetts Institute of Technology, Cambridge, Massachusetts, United States

Abstract

We show that the density μ of the Smith normal form (SNF) of a random integer matrix exists and equals a product of densities μps of SNF over Z/psZ with p a prime and s some positive integer. Our approach is to connect the SNF of a matrix with the greatest common divisors (gcds) of certain polynomials of matrix entries, and develop the theory of multi-gcd distribution of polynomial values at a random integer vector. We also derive a formula for μps and determine the density μ for several interesting types of sets.

*Speaker