A Formula for the Mo bius Function of the Permutation Poset Based on a Topological Decomposition

Jason P Smith*[1]

[1]Department of Computer and Information Sciences [Strathclyde] – University of Strathclyde, 26 Richmond Street, Glasgow G1 1XH, UK, United Kingdom

Abstract

The poset P of all permutations ordered by pattern containment is a fundamental object of study in the field of permutation patterns. This poset has a very rich and complex topology and an understanding of its Mo bius function has proved particularly elusive, although results have been slowly emerging in the last few years. Using a variety of topological techniques we present a two term formula for the Mo bius function of intervals in P. The first term in this formula is, up to sign, the number of so called normal occurrences of one permutation in another. Our definition of normal occurrences is similar to those that have appeared in several variations in the literature on the Mo bius function of this and other posets, but simpler than most of them. The second term in the formula is (still) complicated, but we conjecture that it equals zero for a significant proportion of intervals. We present some cases where the second term vanishes and others where it is nonzero. Computing the Mo bius function recursively from its definition has exponential complexity, whereas the computation of the first term in our formula is polynomial and the exponential part is isolated to the second term, which seems to often vanish. This is thus the first polynomial time formula for the Mo bius function of what appears to be a large proportion of all intervals of P.

*Speaker