Combinatorial descriptions of the crystal structure on certain PBW bases

Ben Salisbury*[1], Adam Schultze*[2], and Peter Tingley*[3]

[1]Department of Mathematics, Central Michigan University, Mount Pleasant, MI – United States
[2]Department of Mathematics and Statistics, State University of New York at Albany, Albany, NY – United States
[3]Department of Mathematics and Statistics [Chicago] – Department of Mathematics and Statistics, Loyola University Chicago, Chicago, IL 60660, USA, United States

Abstract

Lusztig's theory of PBW bases gives a way to realize the crystal $B(\infty)$ for any simple complex Lie algebra where the underlying set consists of Kostant partitions. In fact, there are many different such realizations, one for each reduced expression for the longest element of the Weyl group. There is an algorithm to calculate the actions of the crystal operators, but it can be quite complicated. For ADE types, we give conditions on the reduced expression which ensure that the corresponding crystal operators are given by simple combinatorial bracketing rules. We then give at least one reduced expression satisfying our conditions in every type except E8, and discuss the resulting combinatorics. Finally, we describe the relationship with more standard tableaux combinatorics in types A and D.

*Speaker