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# Symmetric Chain Decompositions and the Strong Sperner Property for Noncrossing Partition Lattices

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## Abstract

We prove that the noncrossing partition lattices associated with the complex reflection groups  $G(d, d, n)$  for  $d, n \geq 2$  admit a decomposition into saturated chains that are symmetric about the middle ranks. A consequence of this result is that these lattices have the strong Sperner property, which asserts that the cardinality of the union of the  $k$  largest antichains does not exceed the sum of the  $k$  largest ranks for all  $k \leq n$ . Subsequently, we use a computer to complete the proof that any noncrossing partition lattice associated with a well-generated complex reflection group is strongly Sperner, thus affirmatively answering a special case of a question of D. Armstrong. This was previously established only for the Coxeter groups of type A and B.

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