
Weak Separation, Pure Domains and Cluster Distance

Miriam Farber*¹ and Pavel Galashin*¹

¹Department of Mathematics [MIT] – Headquarters Office Building 2, Room 236 77 Massachusetts Avenue Cambridge, MA 02139-4307, United States

Abstract

Following the proof of the purity conjecture for weakly separated sets, recent years have revealed a variety of wider classes of pure domains in different settings. In this paper we show the purity for domains consisting of sets that are weakly separated from a pair of "generic" sets I and J . Our proof also gives a simple formula for the rank of these domains in terms of I and J . This is a new instance of the purity phenomenon which essentially differs from all previously known pure domains. We apply our result to calculate the cluster distance and to give lower bounds on the mutation distance between cluster variables in the cluster algebra structure on the coordinate ring of the Grassmannian. Using a linear projection that relates weak separation to the octahedron recurrence, we also find the exact mutation distances and cluster distances for a family of cluster variables.

*Speaker