Toric matrix Schubert varieties and root polytopes (extended abstract)

Laura Escobar*[1] and Karola Mészáros*[2]

[1]Department of Mathematics [Urbana] – Department of Mathematics 1409 W. Green Street Urbana, IL 61801, United States

[2]Department of Mathematics [Cornell] – Cornell UniversityMalot Hall Ithaca, NY 14853-2401, United States

Abstract

Start with a permutation matrix π and consider all matrices that can be obtained from π by taking downward row operations and rightward column operations; the closure of this set gives the matrix Schubert variety $X\pi$. We characterize when the ideal defining $X\pi$ is toric (with respect to a $2n - 1$-dimensional torus) and study the associated polytope of its projectivization. We construct regular triangulations of these polytopes which we show are geometric realizations of a family of subword complexes. We also show that these complexes can be realized geometrically via regular triangulations of root polytopes. This implies that a family of β-Grothendieck polynomials are special cases of reduced forms in the subdivision algebra of root polytopes. We also write the volume and Ehrhart series of root polytopes in terms of β-Grothendieck polynomials. Subword complexes were introduced by Knutson and Miller in 2004, who showed that they are homeomorphic to balls or spheres and raised the question of their polytopal realizations.

*Speaker

sciencesconf.org:fpsac2016:113757