The facial weak order in finite Coxeter groups

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Résumé

We investigate a poset structure that extends the weak order on a finite Coxeter group W to the set of all faces of the permutahedron of W. We call this order the facial weak order. We first provide two alternative characterizations of this poset: a first one, geometric, that generalizes the notion of inversion sets of roots, and a second one, combinatorial, that uses comparisons of the minimal and maximal length representatives of the cosets. These characterizations are then used to show that the facial weak order is in fact a lattice, generalizing a well-known result of A. Bjo rner for the classical weak order. Finally, we show that any lattice congruence of the classical weak order induces a lattice congruence of the facial weak order, and we give a geometric interpretation of its classes.

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