
Non-representable hyperbolic matroids

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Abstract

The generalized Lax conjecture asserts that each hyperbolicity cone is a linear slice of the cone of positive semidefinite matrices. Hyperbolic polynomials give rise to a class of (hyperbolic) matroids which properly contains the class of matroids representable over the complex numbers. This connection was used by the first author to construct counterexamples to algebraic (stronger) versions of the generalized Lax conjecture by considering a non-representable hyperbolic matroid. The *Vamos* matroid and a generalization of it are to this day the only known instances of non-representable hyperbolic matroids. We prove that the Non-Pappus and Non-Desargues matroids are non-representable hyperbolic matroids by exploiting a connection, due to Jordan, between Euclidean Jordan algebras and projective geometries. We further identify a large class of hyperbolic matroids that are parametrized by uniform hypergraphs and prove that many of them are non-representable. Finally we explore consequences to algebraic versions of the generalized Lax conjecture.

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